ON THE STABILITY OF A PULSATING PLASMA CYLINDER

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The present paper concerns the stability of an ideally conductive cylindrically symmetric plasma confined by a time-variable magnetic field. The example of a compressible homogeneous cylinder is used to consider instability analogous to that of a heavy fluid supported against gravity by a lighter medium. The conditions under which nonradial oscillations are excited, are analyzed for an inhomogeneous, periodically compressed and expanded plasma. With small pulsation amplitudes, this instability is of a resonance character.

Plasma behavior is described by means of the usual system of magnetohydrodynamics equations. The principal portion of the computations is carried out for a plasma in a strong external magnetic field (Sections 2, 4, 5). An example of a gravitating cylinder whose radius changes as a result of natural radial oscillations is investigated in Section 6.

1. Basic equations. Let us consider the motions of an ideally conductive nonviscous plasma described by the following system of magnetohydrodynamics equations;

$$\rho \frac{d\mathbf{v}}{dt} = -\nabla p - \frac{1}{4\pi} \mathbf{H} \times \operatorname{rot} \mathbf{H} - \rho \nabla \Phi$$
$$\frac{d\rho}{dt} = -\rho \operatorname{div} \mathbf{v}, \qquad \frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \tag{1.1}$$
$$\frac{\partial \mathbf{H}}{\partial t} = \operatorname{rot} (\mathbf{v} \times \mathbf{H}), \qquad \nabla^2 \Phi = 4\pi G_0 \rho$$

where $\mathbf{v}$ is the velocity, ρ the density, Φ the gravitational potential, and $G_0 = 6.67 \times 10^{-8}$ dyn cm^2/g^2 is the gravitational constant. The gravitation of the medium is taken into account in (1.1), which is important for configurations of interest in astrophysics.

System (1.1) must be supplemented by the energy transfer equation. The form of this equation, however, turns out to be not significant for the problem of stability of a plasma in a strong magnetic field. In studying the stability of a plasma in a magnetic field whose pressure is comparable to that of the plasma, we shall make use of the equation for an adiabatic process

$$\frac{d}{dt} (p\rho^{-\gamma}) = 0 \qquad (\gamma = \text{const}) \tag{1.2}$$

Let us consider cylindrically symmetric motion first. We assume that the plasma can rotate with a uniform angular velocity. The magnetic field has two components. In cylindrical coordinates r, φ, z the distributions are

$$v_r = v_r(r, t), \quad v_\varphi = rW(t), \quad v_z = 0, \quad H_r = 0$$
$$H_\varphi = rg(r, t), \quad H_z = h(r, t) \tag{1.3}$$

We now introduce the Lagrange variables a_0, φ_0, z_0, t_0,

$$a \equiv r_0 = (r)_{t_0=0}, \quad \varphi_0 = (\varphi)_{t_0=0}, \ \ldots$$

$$r = a + \int_0^{t_0} v_r(a, t_0)\, dt_0, \quad \varphi = \varphi_0 + \int_0^{t_0} W(t_0)\, dt_0 \quad \begin{pmatrix} z = z_0 \\ t = t_0 \end{pmatrix} \tag{1.4}$$

Recalling that

$$\frac{\partial}{\partial t} + v_r \frac{\partial}{\partial r} + W \frac{\partial}{\partial \varphi} = \frac{\partial}{\partial t_0}, \quad \frac{\partial}{\partial r} = \chi \frac{\partial}{\partial a}, \quad \frac{\partial}{\partial \varphi} = \frac{\partial}{\partial \varphi_0}$$

$$\chi = \left(\frac{\partial r}{\partial a}\right)^{-1}, \quad v_r = \frac{\partial r}{\partial t_0}$$

from system (1.1),(1.2) we obtain (the subscript 0 of t will henceforth be omitted)

$$\frac{rx}{\chi} = ax_0, \quad x = \rho, g \quad \text{or} \quad h, \quad x_0 = (x)_{t=0}, \quad r^2 W = a^2 W_0 \tag{1.5}$$

$$\frac{a\rho_0}{r}\left(\frac{\partial^2 r}{\partial t^2} - \frac{a^4 W_0^2}{r^3}\right) = -\frac{\partial}{\partial a}\left[p(a, t) + \frac{a^2\chi^2}{8\pi r^2}(h_0^2 + r^2 g_0^2)\right] -$$
$$- \frac{a^2 g^2{}_0 \chi}{4\pi r} - \frac{4\pi a \rho_0 G_0}{r^2}\int_0^a a\rho_0(a)\, da \tag{1.6}$$

$$p\rho^{-\gamma} = p_0 \rho_0^{-\gamma} \quad \text{or} \quad p = p_0 (a\chi / r)^\gamma \tag{1.7}$$

Formula (1.7) is valid for adiabatic motions only. Equation (1.6) defines the function $r(a, t)$. It is necessary here to specify the distributions of density and other quantities at the initial instant. For a plasma cylinder in a vacuum, solution (1.6) must be related to the solution of the exterior problem. The sum of the plasma and magnetic pressures, i.e. the expression in square brackets in the right-hand side of (1.6), must be continuous. The peculiarities of the magnetic field in a vacuum are determined by those external sources which produce the plasma motion under consideration. In the case of natural oscillations of the radial pulsation type, this field vanishes at infinity.

Now let us suppose that a small perturbation is imposed on the radial motion we have been considering, so that the total values of the density, velocity, etc., are $\rho + \rho^*, \mathbf{v} + \mathbf{v}^*$ where $\rho = \rho(r, t), \mathbf{v} = \mathbf{v}(r, t), \ldots$ are determined from Equations (1.5) to (1.7), where the asterisk denotes a perturbation which depends on φ and z as $\exp i(m\varphi + kz)$. Linearizing system (1.1) and converting from the Euler variables to the Lagrange variables introduced above, we obtain

$$\rho\left[\chi \frac{\partial}{\partial t}\left(\frac{1}{\chi} v_r^*\right) - 2Wv_\varphi^*\right] + \rho^*\left(\frac{\partial^2 r}{\partial t^2} - rW^2 + \chi \frac{\partial \Phi}{\partial a}\right) =$$
$$= -\chi \frac{\partial p_\Sigma^*}{\partial a} + \frac{1}{4\pi}(isH_r^* - 2gH_\varphi^*) + \chi \frac{\partial \rho}{\partial a}\Phi^* \tag{1.8}$$

$$\rho\left(\frac{1}{r}\frac{\partial r v_\varphi^*}{\partial t}+2Wv_r^*\right)=-\frac{im}{r}p_\Sigma^*+\frac{1}{4\pi}\left(isH_\varphi^*+\frac{\chi}{r}\frac{\partial r^2 g}{\partial a}H_r^*\right) \quad (1.9)$$

$$\rho\frac{\partial v_z^*}{\partial t}=-ikp_\Sigma^*+\frac{1}{4\pi}\left(isH_z^*+\chi\frac{\partial h}{\partial a}H_r^*\right) \quad (1.10)$$

$$\frac{\chi}{r}\frac{\partial}{\partial t}\left(\frac{r}{\chi}\rho^*\right)=-\rho\operatorname{div}\mathbf{v}^*-\chi\frac{\partial\rho}{\partial a}v_r^* \quad (1.11)$$

$$\frac{1}{r}\frac{\partial rH_r^*}{\partial t}=isv_r^*, \quad \chi\frac{\partial}{\partial t}\left(\frac{1}{\chi}H_\varphi^*\right)=-rg\operatorname{div}\mathbf{v}^*+isv_\varphi^*-\chi r\frac{\partial g}{\partial a}v_r^* \quad (1.12)$$

$$\frac{\chi}{r}\frac{\partial}{\partial t}\left(\frac{rH_z^*}{\chi}\right)=-h\operatorname{div}\mathbf{v}^*+isv_z^*-\chi\frac{\partial h}{\partial a}v_r^* \quad (1.13)$$

$$\frac{\chi}{r}\frac{\partial}{\partial a}\left(r\chi\frac{\partial\Phi^*}{\partial a}\right)-\left(\frac{m^2}{r^2}+k^2\right)\Phi^*=4\pi G_0\rho^* \quad (1.14)$$

$$p_\Sigma^*=p^*+\rho\Phi^*+\frac{1}{4\pi}(rgH_\varphi^*+hH_z^*) \quad (1.15)$$

$$\operatorname{div}\mathbf{v}^*=\frac{\chi}{r}\frac{\partial rv_r^*}{\partial a}+\frac{imv_\varphi^*}{r}+ikv_z^*, \qquad s=mg+kh \quad (1.16)$$

From Equation (1.2) we have

$$\frac{\partial p^*}{\partial t}+\frac{\gamma\chi p^*}{r}\frac{\partial}{\partial a}\left(r\frac{\partial r}{\partial t}\right)=-\gamma p\operatorname{div}\mathbf{v}^*-\chi\frac{\partial p}{\partial a}v_r^* \quad (1.17)$$

In studying the stability of a confined plasma in a vacuum, it is necessary to make use of conditions which make it possible to relate the solutions of the interior and exterior problems. Let us derive such conditions for the boundary layer with its abrupt density jump. Let the density $\rho_0(a)$ in the layer $R-\delta\leqslant a\leqslant R$, $\delta\ll R$, vary from some finite value to zero, and let the pressure $p_0(a)$ be small. We shall assume that the quantities $g, h, r, v_r=\partial r/\partial t, \partial r/\partial a$ and $\partial v_r/\partial a$ are functions of t alone in the layer to within corrections of the order δ/R. This means, in particular, that there are no surface currents (the region with such currents must be considered as belonging to the interior).

We multiply system (1.8) to (1.17) by da/χ and integrate from $R-\delta$ to a, where $R-\delta\leqslant a\leqslant R$. Assuming that

$$\int_{R-\delta}^{R} v_r^*da\sim v_r^*\delta$$

it is the case that

$$(p_\Sigma^*-\rho\Phi^*)|_{R-\delta}^{a}+\frac{1}{\chi}\left(\frac{\partial^2 r}{\partial t^2}-rW^2+\chi\frac{\partial\Phi}{\partial a}\right)\int_{R-\delta}^{a}\rho^*da=0 \quad (1.18)$$

$$\left(\frac{\partial\Phi^*}{\partial a}\right)\Big|_{R-\delta}^{a}-\frac{4\pi G_0}{\chi^2}\int_{R-\delta}^{a}\rho^*da=0 \quad (1.19)$$

$$(\rho v_r^*)|_{R-\delta}^{a}+\frac{1}{r}\frac{\partial}{\partial t}\left(\frac{r}{\chi}\int_{R-\delta}^{a}\rho^*da\right)=0 \quad (1.20)$$

where the integrals of $\mathbf{v}^*, \mathbf{H}^*, p^*$ and p_Σ^* are small quantities,

$$\int_{R-\delta}^{R} \mathbf{H}^* da \sim \mathbf{H}^* \cdot \delta, \text{ etc.}$$

The initial supposition as regards the magnitude of the integral of $v_r{}^*$ is confirmed by integrating (1.18) and (1.20). We also find from (1.19) that the potential Φ^* must be continuous in the layer. Setting $a = R$ in (1.18) to (1.20), we obtain the solution matching conditions sought.

2. The case of a strong field. Let us consider the basic stability equations for a plasma in a magnetic field whose pressure is much larger than that of the plasma, so that

$$h_0^2 \gg 8\pi p_0, \qquad a\,(dh_0^2\,/\,da) \sim 8\pi p_0 \tag{2.1}$$

The gravitation of the medium will not be taken into account. It is known [1 and 2] that under conditions (2.1) and when there is no rotation, magnetohydrodynamic instabilities can occur only in the region of long-wave perturbations, for which

$$k^2R^2 \ll 1, \qquad kh_0 \sim mg_0, \qquad m \neq 0 \tag{2.2}$$

Henceforth we shall assume that condition (2.2) is satisfied, although this limitation is an important one in the case of the problem under consideration. Upon appearance of a radial motion, for example, some of the harmonics may become vibrationally unstable.

We shall likewise assume that $\partial v_r\,/\,\partial t \sim v_r^2\,/\,R$, and that the order of magnitude of $v_r(a,\,t)$ does not exceed the thermal velocity v_T $(v_T \sim \sqrt{p_0/\rho_0})$. It is clear that the rate of change of the perturbations described by system (1.8) to (1.13) is of the same order of magnitude. Under conditions (2.1) and (2.2), it follows from (1.13) that $\operatorname{div}\mathbf{v}^* \approx 0$, where the term containing $v_z{}^*$ in the latter equation is negligibly small. Further, making use of Equations (1.12), we find that

$$v_r{}^* = \frac{1}{isr}\frac{\partial rH_r{}^*}{\partial t}, \qquad v_\varphi{}^* = \frac{\chi}{m}\frac{\partial}{\partial a}\left(\frac{1}{s}\frac{\partial rH_r{}^*}{\partial t}\right), \qquad H_\varphi{}^* = \frac{i\chi}{m}\frac{\partial rH_r{}^*}{\partial a} \tag{2.3}$$

Equation $\operatorname{div}\mathbf{H}^* = 0$ follows from (2.3). Substituting Expressions (2.3) into Equations (1.11), (1.9) and (1.8), we find that

$$\frac{\partial}{\partial t}\frac{r\rho^*}{\chi} = \frac{i}{s}\frac{\partial\rho}{\partial a}\frac{\partial rH_r{}^*}{\partial t} \tag{2.4}$$

$$p_\Sigma{}^* = \frac{i\rho}{m^2}\left\{\frac{\partial}{\partial t}\left[r\chi\frac{\partial}{\partial a}\left(\frac{1}{s}\frac{\partial rH_r{}^*}{\partial t}\right)\right] - \frac{2imW}{s}\frac{\partial rH_r{}^*}{\partial t}\right\} + \frac{ir\chi}{4\pi m^2}\left(s\frac{\partial rH_r{}^*}{\partial a} - \frac{m}{r}\frac{\partial r^2g}{\partial a}H_r{}^*\right) \tag{2.5}$$

$$\chi\rho\left[\frac{\partial}{\partial t}\left(\frac{1}{isr\chi}\frac{\partial rH_r{}^*}{\partial t}\right) - \frac{2W}{m}\frac{\partial}{\partial a}\left(\frac{1}{s}\frac{\partial rH_r{}^*}{\partial t}\right)\right] + \rho^*\left(\frac{\partial^2 r}{\partial t^2} - rW^2\right) + \chi\frac{\partial p_\Sigma{}^*}{\partial a} - \frac{i}{4\pi}\left(sH_r{}^* - \frac{2g\chi}{m}\frac{\partial rH_r{}^*}{\partial a}\right) = 0 \tag{2.6}$$

where $h_0 \approx$ const, and corrections of the order k^2R^2 are omitted.

Knowing the solution of system (2.4) to (2.6), we can determine H_z^* with the aid of the energy transfer equation and (1.15). This fact is no hindrance in finding the solution of the next approximation, since the correction for H_z^* will appear along with the known function H_z^*, and the number of equations will equal the number of unknowns. Due to the fact that the form of the energy transfer equation is not significant, the situation here resembles that of an incompressible medium.

Let us consider the boundary conditions for a plasma cylinder in a vacuum. For $a \geqslant R$, $\mathbf{H}^* = \nabla\psi^*$ and $\nabla^2\psi^* = 0$. Hence, for small k^2R^2,

$$H_r^* = -\frac{im}{|m|} H_\varphi^* = lB_V(t)\, r^{-|m|-1}, \qquad l = \exp i(m\varphi + kz)$$
$$H_z^* = \frac{kr}{m} H_\varphi^*, \quad p^* = 0, \quad \rho^* = 0, \quad B_V = B_V(t) \tag{2.7}$$

In order to determine B_V, we must require fulfillment of the condition of continuity of the normal component of magnetic field perturbation. In the absence of surface currents, we arrive at the continuity condition for H_r^*. Taking into account (1.5), we find from Expressions (1.20) that

$$\left(\frac{\rho_0}{is_0} rH_r^*\right)\Bigg|_{R-\delta}^{a} + \frac{r}{\chi}\int\limits_{R-\delta}^{a} \rho^* da = C^* \tag{2.8}$$

where C^* is independent of t. The choice of C^* is determined by the relationship between the perturbations of the density and of the magnetic field at the initial instant. In the absence of radial compression or expansion of the plasma, for example, the perturbation components are proportional to $\exp i\omega t$, so that $C^* = 0$. The case where $C^* \neq 0$ corresponds to certain forced oscillations.

Setting $C^* = 0$, from Equations (1.18) and (2.8) we have

$$\left\{\frac{i\rho_0}{s_0}\left(\frac{\partial^2 r}{\partial t^2} - rW^2\right) H_r^* + \left(p_\Sigma^* + \frac{isr}{4\pi |m|} H_r^*\right)\right\}_{a=R-\delta} = 0 \tag{2.9}$$

The problem has thus been reduced to that of finding the solution of system (2.4) to (2.6) which satisfies (2.9) and the condition of boundedness at zero.

3. Radial motion of the plasma. For some types of cylindrically symmetric motions, basic equation (1.6) for the function $r(a, t)$ can be reduced to simpler form. Let us consider adiabatic motions with an arbitrary ratio of magnetic and plasma pressures.

In [3 to 7] it is shown that for certain motions with the velocity v_r, which is a linear function of the radius, the variables in Equation (1.6) are separable. Using a dot to denote a derivative with respect to t, we set

$$r = aw, \quad v_r = a\dot{w} \qquad (w = w(t),\ w(0) = 1) \tag{3.1}$$

From Equations (1.5) and (1.7) we obtain

$$\frac{\rho}{\rho_0} = \frac{g}{g_0} = \frac{h}{h_0} = \frac{W}{W_0} = \frac{1}{w^2}, \quad p = p_0 w^{-2\gamma}, \qquad p_0 = (p)_{t=0} \quad \text{etc.} \tag{3.2}$$

Let us consider the case of a homogeneous cylinder with no surface currents, for which

$$\rho_0 = \rho_{00}, \qquad g_0 = g_{00}$$

$$p_0 = p_{00}\left(1 - \frac{a^2}{R^2}\right), \qquad h_0^2 = h_{00}^2\left(1 - \frac{a^2}{R^2}\right) + h_{V0}^2\,\frac{a^2}{R^2} \tag{3.3}$$

Here ρ_{00}, h_{00}, etc., are constants. For the chosen pressure distribution, (1.6) reduces to the ordinary equation

$$w^{\cdot\cdot} = (W_0^2 - \Omega_H^2)\, w^{-3} - (2\Omega_I^2 + \Omega_G^2)\, w^{-1} + \Omega_P^2 w^{-2\gamma+1} \tag{3.4}$$

$$\Omega_H^2 = \frac{h_{V0}^2 - h_{00}^2}{4\pi\rho_{00}R^2}, \qquad \Omega_I^2 = \frac{g_{00}^2}{4\pi\rho_{00}}, \qquad \Omega_G^2 = 2\pi G_0\rho_{00}, \qquad \Omega_P^2 = \frac{2p_{00}}{\rho_{00}R^2}$$

Integrating (3.4), we find that

$$w^{\cdot 2} = (W_0^2 - \Omega_H^2)\,(1 - w^{-2}) - 2\,(2\Omega_I^2 + \Omega_G^2)\ln w + \\ + \frac{\Omega_p^2}{\gamma - 1}\,[1 - w^{-2(\gamma-1)}] + w_0^{\cdot 2}, \quad w_0^{\cdot} = (w^{\cdot})_{t=0} \tag{3.5}$$

In the region $w^{\cdot} \geqslant 0$ we have

$$t = \text{const} + \int \frac{dw}{\sqrt{f(w)}} \tag{3.6}$$

where $f(w)$ is the right-hand side of Equation (3.5). A similar integral can be written for the interval where $w^{\cdot} \leqslant 0$.

Let us consider the extension of the solution just found into the exterior region.

In a vacuum,

$$H_\varphi = w g_V R^2/a, \qquad H_z = h_V \tag{3.7}$$

where g_v and h_v are functions of t alone. The expression in square brackets in the right-hand side of (1.6) must be continuous at the boundary. Since we are only considering states where there are no surface currents, we will have

$$g_V = g_{00}w^{-2}, \quad h_V = h_{V0}\, w^{-2} \tag{3.8}$$

Such motion requires more than a certain distribution of quantities at the initial instant. Also necessary is the fulfillment of a certain relationship between the forces producing compression or expansion of the plasma. For example, if the longitudinal current is not equal to zero at the initial instant, and if the charged particles are displaced as a result of changes in the longitudinal magnetic field, the total current along the filament must be maintained constant in order to prevent the appearance of surface currents. Such a state is not being considered in the present paper.

The resulting solution makes it possible to investigate various periodic and aperiodic motions of a plasma cylinder. Some of the solutions are investigated in [3 to 7].

Let us consider in greater detail the small oscillations near the equilibrium position. When $t = 0$ and there is no radial velocity, let all of the forces be in a balance,

$$W_0^2 - \Omega_H^2 - 2\Omega_I^2 - \Omega_G^2 + \Omega_P^2 = 0 \tag{3.9}$$

We now determine the motion associated with the presence of a small initial radial velocity $v_r(a, 0)$. From (3.4) or (3.5) we have

$$w = 1 + \varepsilon \sin \Omega t + O(\varepsilon^2), \qquad \varepsilon = w_0^{\cdot} / \Omega \tag{3.10}$$

$$\Omega^2 = 3(W_0^2 - \Omega_H^2) - 2\Omega_I^2 - \Omega_G^2 + (2\gamma - 1)\Omega_P^2 \tag{3.11}$$

where ε is a small parameter. Taking into account (3.9), we can rewrite Formula (3.11) as

$$\Omega^2 = 2[(\gamma - 2)(\Omega_H^2 - W_0^2) + (\gamma - 1)(2\Omega_I^2 + \Omega_G^2)] \tag{3.12}$$

If $\Omega^2 > 0$, the motion is oscillatory. Such pulsation may arise as a result of the appropriate vibration of the magnetic field. In the absence of a magnetic field, the pulastions of the gravitating cylinder can be excited by the build-up of the natural radial oscillations of fundamental frequency. The build-up mechanism will not be considered here.

The equations describing the small-amplitude pulsations can be derived from system (1.5) to (1.7) for arbitrary distributions of the density and magnetic field. Setting

$$r = a[1 + \varepsilon(a) \sin \Omega t + \ldots], \qquad |\varepsilon| \ll 1 \tag{3.13}$$

we obtain

$$x = x_0 \left[1 - \frac{1}{a} \frac{da^2\varepsilon}{da} \sin \Omega t + \ldots\right], \qquad x = \rho, g, h \tag{3.14}$$

$$W = W_0(1 - 2\varepsilon \sin \Omega t + \ldots) \tag{3.15}$$

$$\frac{d}{da}\left(p_0 + \frac{h_0^2 + a^2 g_0^2}{8\pi}\right) + \frac{a g_0^2}{4\pi} - a\rho_0 W_0^2 + \frac{1}{a} 4\pi G_0 \rho_0 \int_0^a a\rho_0(a)\, da = 0 \tag{3.16}$$

$$a\rho_0(4W_0^2 - \Omega^2)\varepsilon = \frac{d}{da}\left\{\left(\gamma p_0 + \frac{h_0^2 + a^2 g_0^2}{4\pi}\right)\frac{1}{a}\frac{da^2\varepsilon}{da}\right\} - \frac{a^2}{4\pi}\frac{dg_0^2}{da}\varepsilon + \frac{8\pi}{a} G_0 \rho_0 \varepsilon \int_0^a a\rho_0(a)\, da \tag{3.17}$$

where (3.16) is the equilibrium condition. For ε = const we arrive at Formulas (3.9) to (3.11).

In the case of forced oscillations, the frequency Ω is a specified quantity. For natural oscillations in a constant external magnetic field, the frequency Ω is determined from the condition of no variable magnetic field component outside the plasma, i.e. from the condition $(da^2\varepsilon / da)_{a=R} = 0$.

For adiabatic pulsations of the plasma in the longitudinal field, alone for $\Omega_G \ll \Omega_P$, $\Omega_I = 0$, $W_0 = 0$, ε = const harmonic motion according to (3.12) is possible only for values $\gamma > 2$. From Equation (3.17) we see that in the case of a strong longitudinal field, even a small departure from a linear relationship between the velocity v_r and the radius a alters this result substantially (for $\Omega \sim \sqrt{p_0 / \rho_0 R^2}$ the oscillatory state is possible if the parameter ε differs from a constant by a quantity of order of $8\pi p_0/h_0^2$).

For high-frequency pulsations $(\Omega \gg \sqrt{p_0 / \rho_0 R^2})$ in a strong longitudinal magnetic field $(h_0^2 \gg 8\pi p_0)$ we have from (3.17) the well-known [8 and 9] Equation

$$\frac{1}{a}\frac{d}{da}\left(\frac{1}{a}\frac{da^2\varepsilon}{da}\right) + \frac{4\pi\Omega^2\rho_0}{h_0^2}\varepsilon = 0 \tag{3.18}$$

We note that papers [8 and 9] also deal with the stability of a cylinder in the presence of high-frequency radial pulsations.

4. Stability of a homogeneous cylinder. Let us consider the stability relative to long-wave perturbations $m \neq 0$ of a homogeneous plasma in a strong field for the case of a uniformly distributed longitudinal current. We assume that expressions (3.1),(3.2) and (3.3) are valid for the velocity, pressure, etc., and that conditions (2.1),(2.2) and $\Omega_G \ll \Omega_P$ are fulfilled. In order to investigate the stability, it is necessary to solve system (2.4) to (2.6) under condition (2.9).

In the absence of radial motion, the solution of (2.4) to (2.6) can be obtained from the general formulas of [8]. It is easy to show that the same solution, but with coefficients dependent on t, satisfies system (2.4) to (2.6) for a radially moving plasma. In this manner we obtain

$$rH_r^* = Kla^{|m|}, \quad p_\Sigma^* = iPla^{|m|}, \quad \rho^* = 0, \quad K = K(t), \quad P = P(t) \tag{4.1}$$

$$l = \exp i\left[kz_0 + m\varphi_0 + mW_0 \int_0^t w^{-2}(t)\,dt\right]$$

$$P = \frac{\rho_{00}}{s_{00}\,|m|}\Big\{K^{\cdot\cdot} + \frac{2i}{m}(m^2 - |m|)WK^{\cdot} + \frac{2w^{\cdot}}{w}K^{\cdot} -$$

$$- (m^2 - 2|m|)W^2K + \frac{\Omega_s(m\Omega_s - 2|m|\Omega_1)}{mw^2}K\Big\}$$

$$\Omega_s = \frac{s_{00}}{\sqrt{4\pi\rho_{00}}}, \qquad \Omega_I = \frac{g_{00}}{\sqrt{4\pi\rho_{00}}}, \qquad s_{00} = mg_{00} + kh_{00}$$

We set

$$K = \frac{1}{w}e^qY, \qquad q = -\frac{i}{m}(m^2 - |m|)W_0\int_0^t \frac{dt}{w^2} \tag{4.2}$$

The formula for P then becomes

$$P = \frac{\rho_{00}e^q}{|m|\,s_{00}w}\left[Y^{\cdot\cdot} - \frac{w^{\cdot\cdot}}{w}Y + \frac{W_0^2}{w^4}Y + \frac{\Omega_s(m\Omega_s - 2|m|\Omega_1)}{mw^2}Y\right]$$

From boundary condition (2.9) we have

$$Y^{\cdot\cdot} + \left\{(|m| - 1)\left(\frac{w^{\cdot\cdot}}{w} - \frac{W_0^2}{w^4}\right) + \frac{2\Omega_s(m\Omega_s - |m|\Omega_1)}{mw^2}\right\}Y = 0 \tag{4.3}$$

For adiabatic motions, $w^{\cdot\cdot}$ can be eliminated with the aid of Equation (3.4), whence we have

$$Y^{\cdot\cdot} + \left\{(|m| - 1)\left(\frac{\Omega_P^2}{w^{2\gamma}} - \frac{\Omega_H^2}{w^4} - \frac{2\Omega_I^2}{w^2}\right) + \frac{2\Omega_s(m\Omega_s - |m|\Omega_1)}{mw^2}\right\}Y = 0 \tag{4.4}$$

The equation just derived permits us to compute the perturbation amplitude at any instant, provided Y and $Y^{\cdot}$ at $t = 0$ are known. In the absence of radial motion, $w = 1$, $Y = \exp t\omega t$, where ω is determined from (4.3) or (4.4). In the region of instability, $\omega^2 < 0$. For a nonrotating filament, the boundaries of this region coincide with those obtained in [1 and 2]. It can be shown that for $w = 1,\ \Omega_I = 0,\ k = 0,\ \Omega_s = 0,\ W \neq 0$, (4.4) implies the known formula for the increment of a trough instability of a rotating plasma [10 and 11].

In the case of a cylinder experiencing compression or expansion, the negative coefficient of Y in (4.3) or (4.4) is likewise associated with an increasing perturbation amplitude. For modes $|m| > 1$, the region of instability may not coincide with that prevailing in the absence of radial motion. In particular, for $\Omega_I = 0,\ W = 0$ adiabatic compression of the plasma can be accompanied by the development of instability if

$$\frac{\Omega_H{}^2}{w^4} > \left\{\frac{\Omega_P{}^2}{w^{2\gamma}} + \frac{2\Omega_s{}^2}{w^2\,(|m| - 1)}\right\} \tag{4.5}$$

This instability is similar to that occurring in a heavy fluid confined by a light medium against the action of gravity [12]. The role of the gravitational force is played by the inertial force. The instability arising with radial compression in the case of a thin tubular plasma shell was studied by Harris [13].

For perturbations with sufficiently large m, inequality (4.5) and the formula for the instability rise time τ can be written as

$$h_{V0}{}^2 - h_{00}{}^2 > 8\pi p_{00} w^{2(2-\gamma)} \tag{4.6}$$

$$\tau = \frac{t_c}{\sqrt{|m|}}\left\{\int\limits_0^{t_c}\left(\frac{\Omega_H{}^2}{w^4} - \frac{\Omega_P{}^2}{w^{2\gamma}}\right)^{1/2} dt\right\}^{-1} \tag{4.7}$$

where t_c is the time constant of the process.

For perturbations $m = \pm 1$, the boundaries of the instability region are determined by the inequality $kh_{00}\ (kh_{00} \pm g_{00}) < 0$, which is a familiar result of [1 and 2]. It need merely be noted that the magnitude of the increment increases as $w(t)$ diminishes.

In addition to the above instabilities, it is possible to have other types which vanish in passing to a cylinder of constant radius. In the case of a pulsating cylinder, for example, (4.3) and (4.4) are Hill-type equations, so that conditions for resonance instability may arise.

Let us consider stability with small pulsations when, according to Formula (3.10), $w(t) = 1 + \varepsilon \sin \Omega t + \ldots$, $\varepsilon = \text{const}$. Neglecting quantities of order ε^2, from Equation (4.3) we find that

$$Y^{\cdot\cdot} + \omega^2\,(1 - \varepsilon b \sin \Omega t)\, Y = 0 \tag{4.8}$$

$$\omega^2 = \frac{2}{m}\,\Omega_s\,(m\Omega_s - |m|\,\Omega_1) - (|m| - 1)\,W_0{}^2$$

$$\omega^2 b = \frac{4}{m}\,\Omega_s\,(m\Omega_s - |m|\,\Omega_1) + (|m| - 1)\,(\Omega^2 - 4W_0{}^2)$$

where Ω is the angular frequency of the pulsations, given by (3.11) for adiabatic motions.

$Y(t) = \exp i\omega t$ for $\varepsilon = 0$. If, on the other hand, $\varepsilon \neq 0$, then it is possible for instability to arise in the region corresponding to $\omega^2 > 0$. For small ε, the results of the general theory of equations with periodic coefficients [14] imply that the exponential rise of the function $Y(t)$ is possible in ranges of the frequency ω situated in the neighborhood of resonance frequencies ω_n, where

$$2\omega_n = n\Omega \qquad (n = 1, 2, \ldots) \tag{4.9}$$

In order to investigate the nth order resonance, it is necessary to obtain the solution [14] with consideration of the terms ε^n. With the aid of Equation (4.8), we can study only the resonance $n = 1$.

Assuming that $2\omega = \Omega + O(\varepsilon)$, we attempt to find the solution of (4.8) in the form [14]

$$Y(t) = y(t) \cos [{}^1/_2\Omega t + \vartheta(t)] \quad (|y^{\cdot}| \ll \Omega |y|, \; |\vartheta^{\cdot}| \ll \Omega |\vartheta|) \tag{4.10}$$

The equations for y and ϑ are obtained from the condition that the expansion for $Y(t)$ contain no terms with the difference $2\omega - \Omega$ in the denominator. This yields

$$y^{\cdot} = -\frac{\varepsilon b\omega^2 y}{2\Omega}\cos 2\vartheta, \qquad \vartheta^{\cdot} = \omega - \frac{\Omega}{2} + \frac{\varepsilon b\omega^2}{2\Omega}\sin 2\vartheta \tag{4.11}$$

We set $\qquad \xi = y\cos(\vartheta + {}^1/_4\pi) \quad \eta = y\sin(\vartheta + {}^1/_4\pi)$

Then $\qquad \xi^{\cdot} = -\left(\omega - \frac{\Omega}{2} + \frac{\varepsilon b\omega^2}{2\Omega}\right)\eta, \quad \eta = \left(\omega - \frac{\Omega}{2} - \frac{\varepsilon b\omega^2}{2\Omega}\right)\xi$

Hence

$$\xi = C_1 e^{\varepsilon\Lambda t} + C_2 e^{-\varepsilon\Lambda t}, \qquad \Lambda = \left\{\frac{b^2\omega^4}{4\Omega^2} - \frac{1}{\varepsilon^2}\left(\omega - \frac{\Omega}{2}\right)^2\right\}^{1/2} \tag{4.12}$$

The formula for Λ can also be written as follows:

$$\Lambda = \left\{\frac{b^2\Omega^2}{64} - \frac{1}{\varepsilon^2}\left(\omega - \frac{\Omega}{2}\right)^2\right\}^{1/2} \tag{4.13}$$

Values of ω lying in the interval

$$1 - {}^1/_4\,|\varepsilon b| < 2\omega / \Omega < 1 + {}^1/_4\,|\varepsilon b| \tag{4.14}$$

are associated with oscillatory instability, and the perturbation amplitude increases proportionally as $\exp \varepsilon\Lambda t$. The instability is occasioned by the resonance build-up of the natural oscillation whose frequency in the absence of radial pulsations is ω. The next Section contains a generalization of these results to include the pulsations of an inhomogeneous plasma.

5. Stability of a pulsating inhomogeneous cylinder. Let us consider the stability of an inhomogeneous plasma in a strong magnetic field on which we have superimposed a variable field of small amplitude and frequency $\Omega \sim \sqrt{p_0 / \rho_0 R^2}$.

As was shown in Section 3, the radius $r = a(1 + \varepsilon\sin\Omega t + \ldots)$, where ε is a constant to within corrections on the order of $8\pi p_0/h_0^2$.

We shall also assume that there is no rotation and no surface currents, and that the density on the surface of the cylinder is $\rho_0(R) = 0$. If we confine our attention to long-wave perturbations (conditions (2.1),(2.2), then system (2.4) to (2.6) is valid (we assume that $\Omega_G^2 \ll \Omega_P^2$). After expansion in ε, basic equation (2.6) and condition (2.9) become

$$\frac{\partial}{\partial a} a \left\{\rho_0 \frac{\partial^2}{\partial a \partial t}\left[(1 + 2\varepsilon \sin \Omega t) \frac{\partial X}{\partial t}\right] + \frac{s_0^2}{4\pi} \frac{\partial X}{\partial a}\right\} - \tag{5.1}$$

$$- \frac{m^2}{a}\left\{\rho_0 \frac{\partial}{\partial t}\left[(1 + 2\varepsilon \sin \Omega t) \frac{\partial X}{\partial t}\right] + \frac{s_0^2}{4\pi} X\right\} -$$

$$- \frac{1}{2\pi} m g_0' s_0 X - m^2 \rho_0' \varepsilon \Omega^2 (\sin \Omega t) X = 0$$

$$\left\{s_0^2 a \frac{\partial X}{\partial a} - s_0 (2 m g_0 - |m| s_0) X\right\}_{a=R} = 0 \tag{5.2}$$

$$X = \frac{1}{s_0} r H_r^* e^{-i(m\varphi_0 + kz_0)}, \qquad \rho_0' = \frac{d\rho_0}{da}, \qquad \varepsilon \approx \text{const}$$

In the zeroth approximation (for $\varepsilon = 0$), the solution can be written as

$$X = \sum_{(p)} A_p X_p (a) \cos (\omega_p t + \psi_p), \quad A_p = \text{const}, \quad \psi_p = \text{const} \tag{5.3}$$

Here $X_p(a)$ satisfy (5.2) and Equation (*)

$$\frac{d}{da}\left\{a\left[(s_0^2 - 4\pi\rho_0\omega_p^2) \frac{dX_p}{da}\right]\right\} - \left[\frac{m^2}{a}(s_0^2 - 4\pi\rho_0\omega_p^2) + 2 m g_0' s_0\right] X_p = 0 \tag{5.4}$$

Multiplying (5.4) by $X_q(a)da$ and integrating from 0 to R, we arrive at the orthogonality condition

$$(\omega_q^2 - \omega_p^2) \int_0^R \rho_0 \left[a X_p' X_q' + \frac{m^2}{a} X_p X_q\right] da = 0 \tag{5.5}$$

To find the solution with consideration of errors of order ε, we make use of the method of perturbation theory. We assume the quantities A_p and ψ_p in Equation (5.3) to be functions of t, and also that $\dot{A}_p \sim \varepsilon \omega_p A_p$, $\dot{\psi}_p \sim \varepsilon \omega_p \psi_p$. On substituting series (5.3) into Equation (5.1), we have

$$\sum_{(p)} 2\omega_p \{(\dot{\psi}_p + \varepsilon\omega_p \sin \Omega t) A_p \cos (\omega_p t + \psi_p) +$$

$$+ (\dot{A}_p + \varepsilon \Omega A_p \cos \Omega t) \sin (\omega_p t + \psi_p)\} \left[\frac{d}{da}(a\rho_0 X_p') - \frac{m^2 \rho_0}{a} X_p\right] +$$

$$+ \varepsilon m^2 \Omega^2 \rho_0' (\sin \Omega t) \sum_{(p)} A_p X_p \cos (\omega_p t + \psi_p) = 0$$

Multiplying this equation by $X_q(a)da$ and integrating from 0 to R

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*) For the distribution $\rho_0 = \rho_{00}(1 - a^2/R^2)$, $g_0 = g_{00} = \text{const}$, for example, solution (5.4) can be expressed in terms of the hypergeometric function

$$X_p(a) = a^{|m|} F\left(\frac{1 + |m| + \sqrt{m^2 + 1}}{2}, \frac{1 + |m| - \sqrt{m^2 + 1}}{2}, 1 + |m|, \frac{4\pi\rho_{00}\omega_p^2 a^2}{R^2(4\pi\rho_{00}\omega_p^2 - s_{00}^2)}\right)$$

Substituting this solution into condition (5.2), we obtain the equation for ω_p^2. The value of ω_p^2 are such that the argument of the function $(F)_{a=R}$ lies in the interval between zero and unity. There are both positive and negative ω_p^2.

we arrive at Equation

$$2\omega_q N_q \{2A^{\cdot}_q \sin(\omega_q t + \psi_q) + 2A_q\psi^{\cdot}_q \cos(\omega_q t + \psi_q) +$$
$$+ \varepsilon(\omega_q + \Omega) A_q \sin[(\omega_q + \Omega)t + \psi_q] - \varepsilon(\omega_q - \Omega) A_q \sin[(\omega_q - \Omega)t + \psi_q]\} +$$
$$+ \varepsilon m^2\Omega^2 \sum_{(p)} M_{pq} A_p \{\sin[(\omega_p + \Omega)t + \psi_p] - \sin[(\omega_p - \Omega)t + \psi_p]\} = 0 \quad (5.6)$$

$$N_q = \int_0^R \rho_0 \left(a X_q'^2 + \frac{m^2}{a} X_q^2\right) da, \quad M_{pq} = -\int_0^R \rho_0' X_p X_q da$$

With the aid of system (5.6) we can investigate the problem of the resonance build-up of nonradial oscillations (corresponding to the resonance $n = 1$ of Section 4). We shall study only those oscillations for which $\omega_p^2 > 0$ We can set $\omega_p > 0$

Resonance is possible when the correction terms in Equation (5.6) include those whose frequency is close to ω_q. For example, let $2\omega_q - \Omega = O(\varepsilon)$, and let the expression $\omega_q \pm \omega_p \pm \Omega$ be some distance away from zero for terms with $p \neq q$. Equating the coefficients of $\sin(\omega_q t + \psi_q)$ and $\cos(\omega_q t + \psi_q)$ to zero, we arrive at Equations (4.11) in which y, ϑ, ω and b have been replaced by A_q, $\vartheta_q = \psi_q + (\omega - {}^1/_2\Omega)t$, ω_q and b_q, respectively. Instead of (4.13) we have

$$\Lambda = \left\{\frac{b_q^2\Omega^2}{64} - \frac{1}{\varepsilon^2}\left(\omega - \frac{\Omega}{2}\right)^2\right\}^{1/2}, \qquad b_q = \frac{4m^2 M_{qq}}{N_q} - 2 \qquad (5.7)$$

In the particular case of a homogeneous cylinder with a clearly defined boundary, the system of eigenfunctions $X_p(a)$ is incomplete, and the method of perturbation is, strictly speaking, inapplicable. Nevertheless, Formula (5.7) with allowance for the relation $|m| M_{qq} = N_q$ does lead to a result which coincides with that which follows from Formula (4.13).

Let us investigate the resonance of the combination of two oscillations when $\omega_p + \omega_q - \Omega = O(\varepsilon)$, $p \neq q$. From Equation (5.6) we obtain

$$4\omega_q N_q A = -\varepsilon m^2\Omega^2 M_{pq} A_p \cos[\psi_p + \psi_q + (\omega_p + \omega_q - \Omega)t]$$
$$4\omega_q N_q A_q \psi_q = \varepsilon m^2\Omega^2 M_{pq} A_p \sin[\psi_p + \psi_q + (\omega_p + \omega_q - \Omega)t]$$

and two similar equations with the subscripts p and q interchanged. Taking into account that the functions $X_p(a)$ are determined to within a constant factor, we set $\sqrt{\omega_p N_p / \omega_q N_q} = A_q(0) / A_p(0)$. The described system then reduces to the form (4.11), and $2\vartheta = \psi_p + \psi_q + (\omega_p + \omega_q - \Omega)t$. For the solution which has a region of instability, the amplitudes A_p and A_q are proportional to $\exp \varepsilon\Lambda t$, where

$$\Lambda = \left\{\frac{b_{pq}^2\Omega^2}{64} - \frac{1}{4\varepsilon^2}(\omega_p + \omega_q - \Omega)^2\right\}^{1/2} \qquad b_{pq} = \frac{2m^2\Omega M_{pq}}{\sqrt{\omega_p\omega_q N_p N_q}} \qquad (5.8)$$

In the region of instability

$$-{}^1/_4\Omega\,|\varepsilon b_{pq}| < \omega_p + \omega_q - \Omega < {}^1/_4\,\Omega\,|\varepsilon b_{pq}| \qquad (5.9)$$

It is interesting to note that the solution of system (5.6) under the conditions $\omega_p - \omega_q - \Omega = O(\varepsilon)$, and when ω_p is not close to 2Ω will be always stable. The expression for Λ is obtained by changing the sign in front of ω_q in Formula (5.8), which yields $\Lambda^2 < 0$.

Thus, first-order resonance is possible provided the sum of the two frequencies of the nonradial oscillations (corresponding to the same m and k) or twice the frequency of one of the oscillations is sufficiently close to the frequency of the radial pulsations. Here $m \neq 0$, and if there is no longitudinal current, then $k \neq 0$ as well. The latter condition follows from the fact that all of the frequencies ω_p are equal to zero for $k = 0$, $\sigma_0 \equiv 0$. Hence we see that the location of the instability intervals depends substantially on the distribution of the plasma density and longitudinal current (inasmuch as ω_p is given by Equation (5.4)).

The indicated computation procedure makes possible an obvious generalization of the foregoing inquiry to include the transition of more complex (e.g. wave-type) nonradial motions into various types of natural oscillations. Here the solution of the perturbed equation must be sought in the form of a solution in eigenfunctions of all the coordinates.

For unbounded plasmas, the problems of transition of certain waves into others are examined in [15 and 16]. The authors of [16], for example, derive an instability condition of the same type as for the resonance of a combination of two oscillations considered in the present Section.

6. On the stability of a gravitating cylinder. Consideration of the gravitation of the medium complicates the stability problem substantially. Let us confine ourselves to an investigation of perturbations $k = 0$ for a homogeneous cylinder in the absence of a longitudinal current. We assume that in the initial state of the medium experiencing compression or expansion, the parameter distributions are described by Formulas (3.1) to (3.4) for $g_{00} = 0$, $\Omega_I = 0$. In astrophysics problems, the magnetic field outside the cylinder is usually small, so that $\Omega_H{}^2$ is negative, and the magnetic pressure is comparable to the plasma pressure. In the case of a plasma confined by a magnetic field, the parameter $\Omega_H{}^2$ is larger than zero, and $\Omega_G{}^2 \ll \Omega_p{}^2$.

In the region of homogeneous density, system of Equations (1.8) to (1.17) defining the stability problem has the following exact particular solution:

$$\frac{p^*}{Q} = \frac{P_\Sigma{}^*}{P} = \frac{rv_r{}^*}{V} = -\frac{imrv_\varphi{}^*}{|m|V} = \frac{\Phi^*}{\Psi} = \frac{h_0 H_z{}^*}{4\pi F} = ila^{|m|} \tag{6.1}$$

$$l = \exp\left[im\left(\varphi_0 + W_0 \int_0^t \frac{dt}{w^2(t)}\right)\right], \quad Q = Q(t), \quad P = P(t) \quad \text{etc.}$$

$$P = -\frac{\rho_{00}}{|m|w^2}\left[V^{\cdot} + \frac{i(m^2 - 2|m|)}{mw^2} W_0 V\right] = Q + \frac{1}{4\pi w^2}(F + 4\pi\rho_{00}\Psi) \tag{6.2}$$

$$\begin{gathered} w^2 F^{\cdot} + 2ww^{\cdot}F + imW_0F + \rho_{00}\Omega_H{}^2 w^{-2} V = 0 \\ w^2 Q^{\cdot} + 2\gamma ww^{\cdot}Q + imW_0 Q - \rho_{00}\Omega_p{}^2 w^{-2\gamma} V = 0 \end{gathered} \tag{6.3}$$

Making use of this solution, we can investigate certain types of perturbations from the class of those for which the wave number $k = 0$. As we shall see, the perturbations under consideration are stable if there is no radial motion of the medium. Equations (6.3) imply that $\Omega_p{}^2 F = -\Omega_H{}^2 \omega^{2(\gamma-1)} Q$, and condition (1.20) yields Equation

$$2\Omega_p{}^2 \int_{R-\delta}^{R} \rho^* \, da = l\,[w^{2\gamma-2} R^{|m|-1} Q + C], \qquad C = \text{const}$$

Noting, further, that the perturbation of the gravitational potential outside the cylinder is proportional to $a^{-|m|}$, and that this potential must be continuous on the perturbed surface, with the aid of boundary condition (1.19) we find that

$$2\,|m|\,\rho_{00}\Omega_P{}^2\Psi = -\,\Omega_G{}^2 w^{2\gamma} Q$$

Substitution of the resulting expressions into condition (1.18) yields $C = 0$.

Now, expressing all of the unknowns in terms of $Q(t)$ and substituting them into Equation (6.2), we arrive at a second-order equation for $Q(t)$. If we set $Q = w^{-2\gamma-1} e^{q} Z$, where q is given by Formula (4.2), we have

$$Z^{\cdot\cdot} + (|m|-1)(\Omega_P{}^2 w^{-2\gamma} - \Omega_H{}^2 w^{-4}) Z = 0 \tag{6.4}$$

This equation is a special case of (4.4), although the latter was derived under the condition that $h_0^2 \gg 8\pi p_0$. For a plasma cylinder pulsating with a small amplitude, (6.4) gives

$$\begin{gathered} Z^{\cdot\cdot} + \omega^2 (1 - \varepsilon b \sin \Omega t) Z = 0, \quad \omega^2 = (|m|-1)(\Omega_G{}^2 - W_0{}^2), \quad \varepsilon\Omega = w_0^{\cdot} \\ \omega^2 b = 2(|m|-1)[(\gamma-2)\,\Omega_H{}^2 + \gamma\,(\Omega_G{}^2 - W_0{}^2)] \\ \Omega^2 = 2(\gamma-2)(\Omega_H{}^2 - W_0{}^2) + 2(\gamma-1)\,\Omega_G{}^2 \end{gathered} \tag{6.5}$$

in which equilibrium condition (3.9) is taken into account. An equation of the type (6.5) was analyzed in Section 4.

Our results concerning the stability of a gravitating cylinder can be applied to the qualitative examination of the stability of spherically symmetric gravitating configurations which play an important role in astrophysics. There is an analogy between the perturbations $k = 0$ for a cylinder and arbitrary perturbations for a sphere. In particular, Equation (6.4) for $\Omega_H = 0$ has a character similar to that of Equation (15) of [17], which deals with the stability of a pulsating homogeneous sphere in relation to perturbations with monotonous dependence on the radius.

Making use of this analogy, we conclude that resonance build-up of nonradial oscillations is indeed possible in a pulsating gravitating sphere of nonuniform density. Resonance of the type (4.9) is studied in [17]. Let us now consider the resonance when where ω_q and ω_p are the frequencies of nonradial oscillations with the same dependence on the angular variables, and Ω is the frequency of the radial pulsations of the sphere. The combination may resonate when some frequency ω_q is sufficiently close to the frequency Ω, and when the difference $\Omega - \omega_q$ is larger than zero. This is because the spectrum of frequencies ω_p always contains infinitesimal frequencies ω_p (we are referring to various nonradial oscillations with a specified dependence on the angular variables) [18]. In this example of combination resonance, the excited oscillations have a frequency close to that of the pulsations, which may lead to beating. Beats in the luminosity curves of variable stars are a common occurance [18]. The described instability mechanism may be the cause behind this effect.

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